

An interpretation for the entropy of a black hole

Baocheng Zhang*

*State Key Laboratory of Magnetic Resonances and Atomic and Molecular Physics,
Wuhan Institute of Physics and Mathematics,
Chinese Academy of Sciences, Wuhan 430071, China and
Graduate University of Chinese Academy of Sciences, Beijing 100049, China*

Qing-yu Cai†

*State Key Laboratory of Magnetic Resonances and Atomic and Molecular Physics,
Wuhan Institute of Physics and Mathematics,
Chinese Academy of Sciences, Wuhan 430071, China*

Ming-sheng Zhan

*State Key Laboratory of Magnetic Resonances and Atomic and Molecular Physics,
Wuhan Institute of Physics and Mathematics,
Chinese Academy of Sciences, Wuhan 430071, China and
Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China*

Li You

Department of Physics, Tsinghua University, Beijing 100084, China

Abstract

We investigate the meaning of the entropy carried away by Hawking radiations from a black hole. We propose that the entropy for a black hole measures the uncertainty of the information about the black hole forming matter's precollapsed configurations, self-collapsed configurations, and inter-collapsed configurations. We find that gravitational wave or gravitational radiation alone cannot carry all information about the processes of black hole coalescence and collapse, while the total information locked in the hole could be carried away completely by Hawking radiation as tunneling.

Key Words: Black hole, entropy, Hawking radiation, tunneling

*Electronic address: zhangbc@wipm.ac.cn

†Electronic address: qycal@wipm.ac.cn

Information is physical and thus it is conserved during any physical process. Information cannot be simply created out of nothing or disappeared into a sink hole. The revolution of information science has elevated the above principle into the status of the fundamental laws of nature [1–3]. This law can be embodied both in classical mechanics and in quantum mechanics. Classically, a physical state is specified by its distribution function in the multi-dimensional phase space for all its degrees of freedom. Liouville’s theorem: the conservation of phase space volume, gives rise to the conservation of entropy or information under Hamiltonian dynamics. In quantum mechanics, the conservation of information is expressed as the unitarity evolution for a quantum system, which implies a pure state will evolve into another pure state, and will never evolve into a mixed state except under interventions from the external world. In general, the conservation of information is viewed as the conservation of entropy conservation quantitatively, *i.e.*, the evolution of an isolated closed system will not lead to entropy increase or information loss.

The discovery of Hawking radiation from a black hole [4, 5], however, brings up a serious challenge to the conservation law of information [6]. It was shown that Hawking radiations governed by a thermal emission causes an increase of entropy after evaporation of a black hole [7]. In other words, information is lost during the process of black hole evaporation. A revisit of the original treatment for Hawking radiation, however, revealed that the background geometry was considered as fixed without enforcing the energy conservation. Including the energy conservation, Parikh and Wilczek obtained a non-thermal spectrum for Hawking radiation due to tunneling [8]. Along this line, we discovered the existence of correlations among Hawking radiations by using the standard statistical method and quantum information theory [9, 10]. By counting the entropy carried away by emitted particles with themselves, we showed the Hawking radiation as tunneling is an entropy conserved process. Thus information remains conserved even during the Hawking radiation from a black hole. Moreover, along our line in Ref.[9, 10], the effect of quantum correction and back reaction for the resolving the information loss paradox has been investigated [11]. It is noted that recently the unitary character of the black hole evolution is shown from the Schmidt decomposition viewpoint [12].

In this paper, we investigate the meaning of the entropy carried away by Hawking radiations from a black hole. Based on the conservation of entropy as we have discussed before [9, 10], we also hope to understand the meaning of the initial black hole entropy. Histor-

ically, many interpretations have been suggested [13–19] to explain the entropy of a black hole entropy, including some novel and profound ideas. Our explanation comes from the perspective of quantum information and our recent analysis of Hawking radiation. We interpret entropy as the uncertainty about the information of the black hole forming matter’s precollapsed configurations, self-collapsed configurations, and inter-collapsed configurations. The explanation is applied to several circumstances, including the formation of a black hole, black hole coalescence, and a common matter dropped into a black hole.

Within the framework of Parikh and Wilczek treatment of Hawking radiation as tunneling [8], the tunneling probability is found to be nonthermal and given by

$$\Gamma(M; E) \sim \exp \left[-8\pi E \left(M - \frac{E}{2} \right) \right]. \quad (1)$$

Quite straightforwardly, the exponential part can be considered as the entropy change of a black hole, $\Delta \mathcal{S} = -8\pi E (M - E/2)$. The negative sign represents the decrease of the black hole entropy associated with each emission. This implies information is carried away by Hawking radiation because a reduced entropy implies a reduced uncertainty or the gaining of information. According to information theory, the entropy carried away by emitted particles is defined by $S(E) = -\ln \Gamma(M; E) = +8\pi E (M - E/2)$, where the positive sign represents an increase of the entropy for the environment surrounding a black hole. We adopt a notation using \mathcal{S}_m and $S(E)$ to denote the entropies respectively for a black hole of mass m and for a radiation of energy E . Thus for the complete system of a black hole plus its Hawking radiations, entropy is not changed during the whole process, although information is indeed carried to the outside a black hole.

At first, we will explain why the tunneling particle could take the entropy by itself in Hawking radiation as tunneling. The most important reason is that the emission process is probabilistic, not deterministic. For each tunneling emission from a black hole with the mass M , we only know a radiation may occur with a probability $\Gamma(M; E)$, nothing else. In other words, the uncertainty of the event (for a radiation with energy E) or the potential information we can gain from the event is $S(E) = -\ln \Gamma(M; E)$.

Proceeding with an explicit presentation, we rewrite the entropy [9] carried away by the emitted particle with an energy E as

$$S(E) = 8\pi E \left(M - \frac{E}{2} \right) = 8\pi E (M - E) + 4\pi E^2, \quad (2)$$

which depends not only on the energy of the emitted particle, but also on the mass of the black hole. After the emission of a particle with an energy E , the whole system consists of a new black hole with mass $M - E$ and an emission with energy E . The first term in the Eq. (2) is identical in form to the correlation between the black hole and the particle. The second term in Eq. (2) is the familiar entropy of a black hole with mass or energy E . This understanding can be further clarified by considering the coalescing of two black holes.

We next consider two Schwarzschild black holes with respective mass M and m . Their respective entropies are $4\pi M^2$ and $4\pi m^2$. Assuming they are a large distance apart initially and are being held stationary, both the total kinetic energy and momentum can be taken as zero. Due to gravitational attractive interaction, the two black holes will approach each other with ever increasing velocity until they experience a head-on collision. If the collision is elastic, the total kinetic energy and momentum will be conserved, the two black holes will not coalesce into one larger black hole. The above picture is thus not in conflict with any conservation laws of fundamental physics, such as energy conservation, momentum conservation, and entropy conservation. But it is not a realistic scenario as two colliding black holes will form a larger black hole. While energy conservation and momentum conservation is strictly held, entropy is not conserved when two black holes coalesce into one as we can easily check by writing down the entropy of the resulting black hole as

$$\mathcal{S}_{M+m} = 4\pi(M + m)^2 = 4\pi M^2 + 4\pi m^2 + 8\pi Mm. \quad (3)$$

The extra (third) term $8\pi Mm$ measures some kind of correlation generated by gravitational interactions. Before the two coalesce, this correlation constitutes of actual information describing dynamics due to gravitational force. It can be gained by an exterior observer, so the entropy for the whole system will not change. After the new black hole forms, the correlation is covered by the resulting event horizon, the exterior observer will not be able to obtain this information about correlation, so that entropy increases, or the uncertainty for the new system (the new black hole) increases.

On the other hand, when the two black holes collide and coalesce into one, gravitational waves are emitted. Is it possible that the gravitational radiations actually carry away the amount of information corresponding to the increased entropy? If one takes an affirmative attitude towards this question, the entropy carried away by gravitational radiations has to

be at least of the following magnitude

$$\begin{aligned} S(m') &= \mathcal{S}_M + \mathcal{S}_m - \mathcal{S}_{M+m-m'} \\ &= 8\pi(M+m)m' - 4\pi m'^2 - 8\pi Mm, \end{aligned} \quad (4)$$

where m' is the energy of gravitational wave radiation.

According to the classical area theorem [20], when two black holes coalesce, the area of the final event horizon is greater than the sum of the areas of the initial horizon. Thus, the entropy for the new black hole will be greater than the sum of the entropies for the two initial black holes, because the entropy for a black hole is proportional to its area of the event horizon. This gives the following inequality $\mathcal{S}_{M+m-m'} > \mathcal{S}_M + \mathcal{S}_m$, from which, we find $S(m') < 0$. Gravitational radiation thus cannot carry away all the increased entropy. There must exist other correlations covered by the event horizon of the coalesced black hole and inaccessible to the exterior observers. This leads to the increase of entropy. In other words, gravitational wave radiations alone cannot carry all the information about the gravitational interactions during the collapse.

More generally, we consider a common matter of mass m falling into a black hole. The entropy for the resulting black hole can be expressed as $\mathcal{S}_{M+m} = 4\pi(M+m)^2 = 4\pi M^2 + 4\pi m^2 + 8\pi Mm$ due to conservation of energy. If the initial entropy of the fallen matter is $\mathcal{S}^{(0)}$, the net entropy increase is

$$\Delta\mathcal{S} = 4\pi m^2 + 8\pi Mm - \mathcal{S}^{(0)}. \quad (5)$$

Without the detailed knowledge for the microstate of the fallen mass, it is impossible to estimate its entropy. However, the expression for the entropy change (5), suggests the description of the process for a matter falling into a black hole can be generally separated into two stages: 1), the fallen matter becomes a black hole in a self-collapsed process. This is analogous to how a common mass m would collapse into a black hole. The entropy increase reveals the inaccessible information about the collapse. In quantitative terms, this increased entropy is given by $4\pi m^2 - \mathcal{S}^{(0)}$; 2), the initial black hole and the black hole of the fallen mass coalesce into a new black hole in an inter-collapsed process. The process is always accompanied by the emissions of gravitational waves. Hawking once obtained an upper bound of 29% for the total energy of gravitational waves emitted when one collapsed object captures another [21]. Recently this upper bound has reduced appreciably based

on numerical simulations of the Einstein's equation [22]. The previous considerations as discussed above show that gravitational wave radiations cannot carry away all the increased entropy. Therefore this inter-collapsed process leads to an increase of entropy due to the inaccessible information about the correlations during the coalescence and collapse. This increased quantity is exactly given by $8\pi Mm$.

The above general case for a common matter falling into a black hole also applies to the case of a common matter collapsing into one black hole. We can simply view this process as first from individual parts of the common matter forming individual baby black holes; the baby black holes interact and merge with each other and finally coalesce and form a new larger black hole.

In the following we proceed to investigate the meaning of the entropy (2). Before the formation of a black hole, we denote the entropy for a particle (mass) with energy E by $\mathcal{S}^{(0)}$, the entropy (2) is then conveniently reexpressed as

$$S(E) = 8\pi E(M - E) + (4\pi E^2 - \mathcal{S}^{(0)}) + \mathcal{S}^{(0)}. \quad (6)$$

Thus this entropy, which measures the information carried away by the tunneling particle, measures respectively its inter-collapsed configurations, self-collapsed configurations, and the precollapsed configurations. In the radiation process, in addition to the information or entropy $\mathcal{S}^{(0)}$, inherent to the radiating particle, the correlation between the radiation and the remaining black hole $8\pi E(M - E)$, generated from the inter-collapsed process, and entropy of the remaining black hole $(4\pi E^2 - \mathcal{S}^{(0)})$, generated by the self-collapsed process, are carried away as well.

The exterior correlations include correlations of all emitted particles with each other, which can be shown for any queue of Hawking radiations as sequential tunneling. For this purpose, we consider a queue of emissions ordered according to E_1, E_2, \dots, E_{n-1} . The entropy of the first emission with an energy E_1 is

$$S(E_1) = 8\pi E_1(M - E_1) + (4\pi E_1^2 - \mathcal{S}_1^{(0)}) + \mathcal{S}_1^{(0)},$$

where the term $8\pi E_1(M - E_1)$ includes all the correlations between the particle with energy E_1 and all other particles with energies E_2, \dots, E_{n-1}, E_n . Given the first emission with an

energy E_1 , the entropy of the second emission with an energy E_2 is

$$S(E_2|E_1) = 8\pi E_2(M - E_1 - E_2) + (4\pi E_2^2 - \mathcal{S}_2^{(0)}) + \mathcal{S}_2^{(0)}. \quad (7)$$

As before, this entropy is partitioned into three terms with $\mathcal{S}_2^{(0)}$ referring to the precollapsed configurations, $(4\pi E_2^2 - \mathcal{S}_2^{(0)})$ about self-collapsed configuration, and the correlation, or partial information $8\pi E_2(M - E_1 - E_2)$ about inter-collapsed configuration. The information about the correlations between the first emission E_1 and the second emission E_2 is already carried out by the first emission. Therefore for the second emission (7), its correlation with the first $8\pi E_1 E_2$ must be subtracted. Analogously, for the third emission with energy E_3 ,

$$S(E_3|E_1, E_2) = 8\pi E_3(M - E_1 - E_2 - E_3) + (4\pi E_3^2 - \mathcal{S}_3^{(0)}) + \mathcal{S}_3^{(0)}. \quad (8)$$

It is easy to check that correlations between the third emission E_3 and first one E_1 , and between the third emission E_3 and the second emission E_2 are already subtracted. We summed together, $S(E_1) + S(E_2|E_1) + S(E_3|E_1, E_2)$ contains no redundant information or entropy. Thus, this sum of entropies is equivalent to the reduced entropy for the black hole, or

$$\begin{aligned} S(E_1) + S(E_2|E_1) + S(E_3|E_1, E_2) \\ = 4\pi(M - E_1 - E_2 - E_3)^2 - 4\pi M^2 \\ = \Delta S_{\text{BH}}. \end{aligned} \quad (9)$$

This step by step construction shows that the Hawking radiations carry with themselves information, in fact, all information, because no information loss, or entropy increase is found. Additionally, our analysis above provides a self-consistent interpretation for the entropy of a black hole according to the information of entropies taken out by the Hawking radiations. The entropy for a black hole merely implies that for an exterior observer, there exists uncertainties for the information about precollapsed configurations, self-collapsed configurations, and inter-collapsed configurations. When a black hole radiates, all these associated information are leaked out through the particles and the correlations between particles.

Repeating the above process of step by step analysis of each Hawking emissions until the black hole is completely exhausted, the entropy or information conservation is found to be

preserved at all times. For a Schwarzschild black hole, however, it is difficult to describe the final emission E_n , whose entropy is

$$S(E_n|E_1, E_2 \cdots, E_{n-1}) = 4\pi E_n^2,$$

which is precisely the same as for a black hole with mass or energy E_n . This shows the final emission is really equivalent to emit itself, because when the black hole is about to vanish due to evaporation, the temperature becomes very high enough to emit the particle with any mass or energy. Moreover, noted that the final black hole could be regarded as a fundamental particle [23, 24], which is stable and emit no radiation. In Ref. [10], when quantum gravity effects [25] are considered, the black hole will evolve into a remnant and the problem of an infinite temperature is voided. Whichever case happens, it seems that our conclusion of entropy conservation in the Hawking radiation process is unaffected.

In conclusion, based on carefully analyzing the entropies carried away by tunneling particles, we find the black hole entropy contains three parts: respectively associated with the information for precollapsed configurations, self-collapsed configurations, and inter-collapsed configurations. All information are covered by the event horizon of a black hole and are inaccessible to exterior observers. When a black hole emits, all such information are taken out of the black hole by the radiations, and this implies that the black hole evaporation is a unitary process.

When two black holes coalesce to form a new black hole, the gravitational waves emitted during the process are found to be incapable of carrying away information associated with the increased entropy. This implies one cannot obtain all the information about the collapsing process by gravitational radiations or gravitational waves. It sounds disappointing. However, our work suggest that Hawking radiations, on the other hand, contain all information about the gravitational collapse.

A final comment concerns the following question: why the entropy of an ordinary matter, which could essentially take any value, changes into a fixed value after fallen/changed into a black hole of the equal energy? As pointed out by some physicists [26–29], black holes have the maximum possible entropy of any object of equal size and this makes them likely end points of all entropy-increasing processes. We don't attempt to prove it or provide an our answer in this paper. Instead, we simply provide an explanation for the increased entropy, which we feel suggests that information about gravitational interaction or grav-

itational spacetime is closed inside the event horizon. This shows that the guess of the maximum entropy is at least not in conflict with information conservation. Of course, a clearer explanation about black hole entropy need a better description for the state of the inner black hole. Although string theory and many other quantum gravity theories can give some such descriptions, the price paid includes additional elements not completely falsifiable at the present stage.

This work is supported by National Basic Research Program of China (NBRPC) under Grant No. 2006CB921203.

-
- [1] R. Landauer, *Information is Physical*, Phys. Today **44**, 23 (1991)
 - [2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge University Press, Cambridge, UK, 2000).
 - [3] L. Susskind and J. Lindesay, *Black Hole, Information and the String Theory Revolution*, (World Scientific Publishing Co. Pte. Ltd., Danvers, USA, 2005)
 - [4] S. W. Hawking, *Black hole explosions?* Nature (London) **248**, 30 (1974).
 - [5] S. W. Hawking, *Particle Creation by Black Holes*, Commun. Math. Phys. **43**, 199 (1975).
 - [6] S. W. Hawking, *Black hoels and thermodynamics*, Phys. Rev. D **14**, 2460 (1976)
 - [7] W. H. Zurek, *Entropy Evaporated by a Black Hole*, Phys. Rev. Lett. **49**, 1683 (1982)
 - [8] M. K. Parikh and F. Wilczek, *Hawking Radiation As Tunneling*, Phys. Rev. Lett. **85**, 5042 (2000).
 - [9] B. Zhang, Q. Y. Cai, L. You and M. S. Zhan, *Hidden Messenger Revealed in Hawking Radiation: a Resolution to the Paradox of Black Hole Information Loss*, Phys. Lett. B, 675, 98 (2009)
 - [10] B. Zhang, Q. Y. Cai, M. S. Zhan, and L. You, *No Information Is Lost: a Revisit of Black Hole Information Loss Paradox*, (arXiv: 0906.5033).
 - [11] D. Singleton, E. C. Vagenas, T. Zhu, and J. R. Ren, *Insights and possible resolution to the information loss paradox via the tunneling picture*, (arXiv: 1055.3778).
 - [12] E. D. Belokolos and M. V. Teslyk, *Scalar field entanglement entropy of a Schwarzschild black hole from the Schmidt decomposition viewpoint*, Class. Quantum Grav. **26** 235008 (2009)
 - [13] R. M. Wald, *Black hole entropy is the Noether charge*, Phys. Rev. D **48**, R3427 (1993)

- [14] C. Callen and F. Wilczek, *On geometric entropy*, Phys. Lett. B **333**, 55 (1994)
- [15] J. Jing, *Thermodynamics of stationary axisymmetric Einstein-Maxwell dilaton-axion black hole*, Nucl. Phys. B **476**, 548 (1996)
- [16] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, Phys. Lett. B **379**, 99 (1996)
- [17] S. Liberati and G. Pollifrone, *Entropy and topology for gravitational instantons*, Phys. Rev. D **56**, 6458 (1997)
- [18] T. K. Won, J. O. John, and Y.-J. Park, *Entropy of the Randall-Sundrum black brane world in the brick-wall method*, Phys. Lett. B **512**, 131 (2001)
- [19] R. V. Buniy and S. D. H. Hsu, *Entanglement entropy, black holes and holography*, Phys. Lett. B **644**, 72 (2007)
- [20] J. M. Bardeen, B. Carter, and S. W. Hawking, *The Four Laws of Black Hole Mechanics*, Commun. Math. Phys. **31**, 161 (1973).
- [21] S. W. Hawking, *Gravitational Radiation from Colliding Black Holes*, Phys. Rev. Lett. **26**, 1344 (1971).
- [22] U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, and J. A. González, *High-Energy Collision of Two Black Holes*, Phys. Rev. Lett. **101**, 161101 (2008).
- [23] G. 't Hooft, *On the quantum structure of a black hole*. Nucl. Phys. B, **256**, 727 (1985)
- [24] Y. K. Ha, *Are Black Holes Elementary Particles?*, arXiv: 0906.3549 [gr-qc]
- [25] M. Arzano, A. J. M. Medved, and E. C. Vagenas, *Hawking radiation as tunneling through the quantum horizon*. J. High Energy Phys. **0509**, 037 (2005)
- [26] J. D. Bekenstein, *Universal upper bound on the entropy-to-energy ratio for bounded systems*, Phys. Rev. D **23**, 287 (1981)
- [27] L. Susskind, *The world as a hologram*, J. Math. Phys. **36**, 6377 (1995)
- [28] R. Bousso, *A covariant entropy conjecture*, JHEP **07**, 004 (1999)
- [29] E. E. Flanagan, D. Marolf, and R. M. Wald, *Proof of classical versions of the Bousso entropy bound and of the generalized second law*, Phys. Rev. D **62**, 084035 (2000)